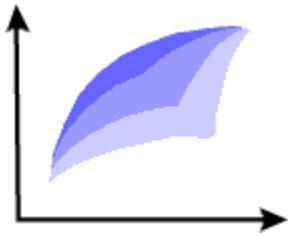
Efficient Frontier



An Online Journal of Practical Asset Allocation

Edited by William J. Bernstein

September 1996

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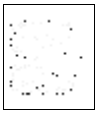

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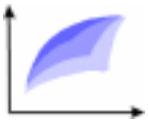

Home Edited by William J. Bernstein Cover

The Basics of Investing and Portfolio Theory

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Efficient Frontier is aimed at investors who are familiar with the basics of finance and portofolio theory. For those who are not, and wish to pursue this area further, the following suggestions are offered:

Investing competence does not come without some sweat. Expect to put at least as much time, effort and intellectual energy into this task as you would for a difficult college course. Four basic areas must be mastered, in the following order:

A. You must acquire a working knowledge of the nature, returns, and risk of various asset classes.

B. You must then acquire an appreciation of how various assets behave in diversified portfolios. This is called portfolio theory. The central concept of portfolio theory is that of the "efficient frontier." This involves identifying the portfolio composition(s) that provide one with the maximum return for a given degree of risk (or alternatively, the least amount of risk for a given return).

C. You must then develop a coherent and well defined personal strategy for the allocation of your assets among broad asset classes (i.e., foreign and domestic stocks and bonds, small versus large, growth versus value stocks, cash, real estate, precious metals, etc.).

D. Finally, you must implement this strategy through the proper choice of investment vehicles (individual stocks and bonds, mutual funds). Here are some simple portfolio illustrations. They are not intended as recommendations:

1. Stone simple: 50% S&P 500 (Vanguard index trust 500) and 50% bonds (Vanguard Bond Index Fund.)

2. Slightly more complex and diversified: 25% each S&P 500, bonds, small stocks (Vanguard small stock index), and foreign stocks (one third each Vanguard European, Pacific, and Emerging Markets Index funds).

3. Highly complex and diversified: A fixed mix of U.S. stocks, bonds, and foreign stocks from #2 above, plus foreign bonds, foreign small stocks, US small value stocks, REITs, precious metals, natural resources, utilities, junk bonds, etc.

The following bibliography provides a start towards accomplishing the above goals. These four books should be read in order:

1. *A Random Walk Down Wall Street*, Burton Malkiel, Norton Publ. (about $15 in paperback).

2. *Asset Allocation*, Roger Gibson, Business One Irwin (about $35).

3. *Global Investing*, Roger Ibbotson and Gary Brinson, McGraw Hill (about $40, also available through Ibbotson Associates).

4. *Value Averaging*, Michael Edelson, IPC ($22.95).

If your time and resources are limited, you can "get by" with the Gibson book alone. Unfortunately, this is also the driest and least well written of the four. The other three books are actually quite pleasant and interesting reads. There is alot of math in all four, which can be discouraging. Simply ignore what you do not understand. The remainder will reward you. An appreciation of the dangers of growth stocks is essential. A good start can be had from a pamphlet from the Tweedy Browne mutual fund group, *What Has Worked in Investing*. Those whose appetites have been whetted can proceed to *The Intelligent Investor* (Harper and Row) by Benjamin Graham. If you wish a lucid explanation of the mathematics of mean variance optimization, try Harrry Markowitz' *Portfolio Selection*.

Investment competence is a lifelong learning process. It helps to subscribe to some periodicals. I highly recommend the *Wall Street Journal*. This is a lot of reading, so pick and choose only what interests you. The first section is an excellent national newspaper, and the *Your Money Matters* weekly feature in the last section provides an excellent review of investment technique. Also, join the American Association of Individual Investors and get a free subscription to the *AAII Journal*, which is another valuable source of general investment info.

A few words are necessary concerning Lou Rukeyser's *Wall Street Week*, the most widely followed piece of financial media. To quote Bernard Baruch, "something that everyone knows isn't worth knowing." Studies show that your long term investment returns will be almost entirely dependent on your allocation of assets among broad asset categories. Mr. Rukeyser instead concentrates almost exclusively on stock picking and market timing. To the unsophisticated investor this may seem useful. In fact, however, long term success in the former is rare, in the latter nonexistent. It is mathematically impossible for the thirty million viewers of this show to beat the market, since they are the market. The same applies to *Barrons, Forbes, Money, Kiplinger's*, and all newsletters. It goes double for Dan Dorfman. (As an example, *Forbes* has a highly respected mutual fund ranking system. Unfortunately, academic analysis has shown that the future performance of its best rated "honor roll" funds is slightly less than average. To cite another remarkable example, a recent National Bureau of Economic Research working paper analyzed the performance of over 200 newsletters, and found that none reliably beat the market on a risk adjusted basis, although many underperformed it with astonishing regularity, including one which lost money at a 5% compounded rate for over 20 years. This is a remarkable accomplishment for the 1974-94

period.)

There is a wealth of inexpensive and powerful investment software available. For $99 per year Morningstar will provide you with quarterly updates of a program which will allow you to screen, sort, compare, rank, and display mutual funds using dozens of criteria. This program also provides a quick and easy way to follow the valuations of many foreign and domestic sectors, i.e, to determine what's cheap and what's overpriced. (This is accomplished by looking at the price/book and price/earnings ratios of the Vanguard Index, Dreyfus Wilshire Target, and DFA foreign and domestic small cap index funds.) For a few dollars more they will also sell you a program to do the same with individual stocks. Last, and most certainly least, is the web. It is difficult to comprehend how so much garbage has found its way into such a small corner of cyberspace. Do not waste your time. To be sure, there is a great wealth of data out there, but this is not of much use to the novice investor. One of the founding fathers of modern portfolio theory, William Sharpe, has both a fascinating homepage and a nearly impenetrable textbook in progress on the web. The only worthwhile educational piece I've found is Frank Armstrong's **Investing for the 21st Century**. This is a well written, highly entertaining, but somewhat superficial tour of basic finance, portfolio theory, and in particular the pitfalls of modern investing. It will reinforce what you get from the above sources.

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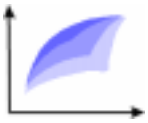

The Coward's Portfolio -- A Modest Proposal

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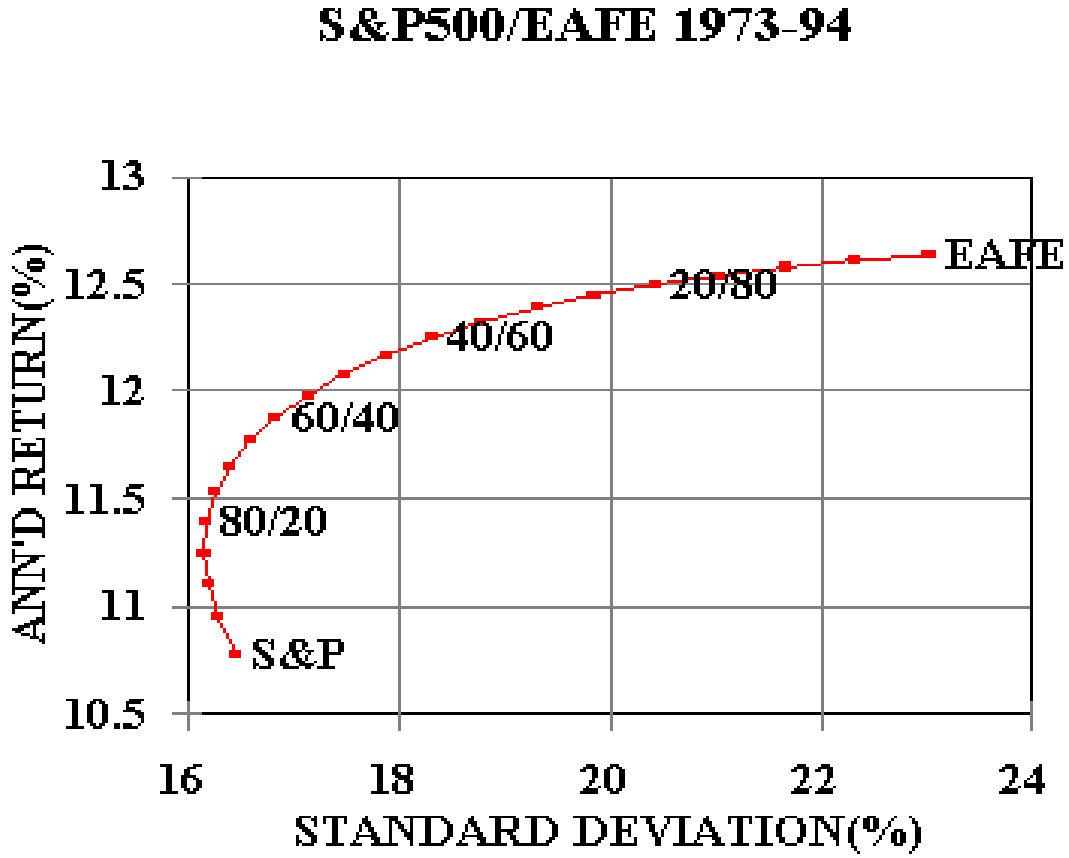
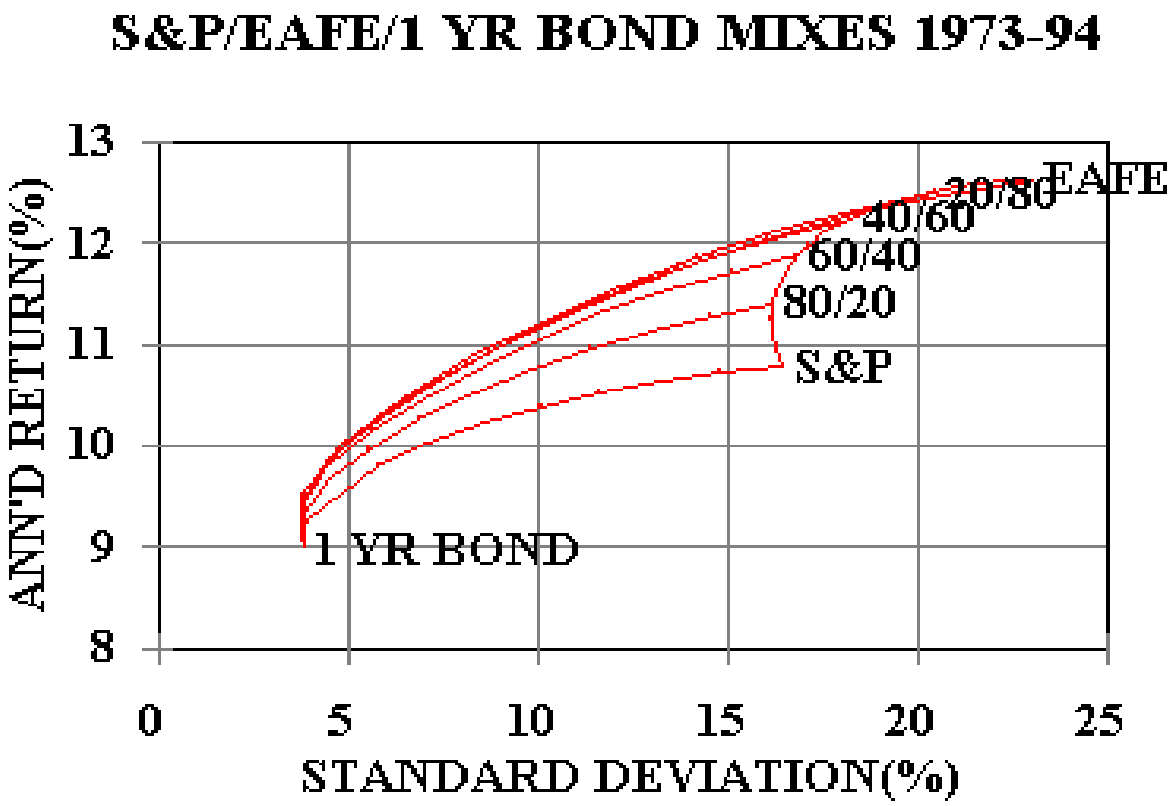
"It's very hard to make predictions, particularly about the future" -- Yogi Berra

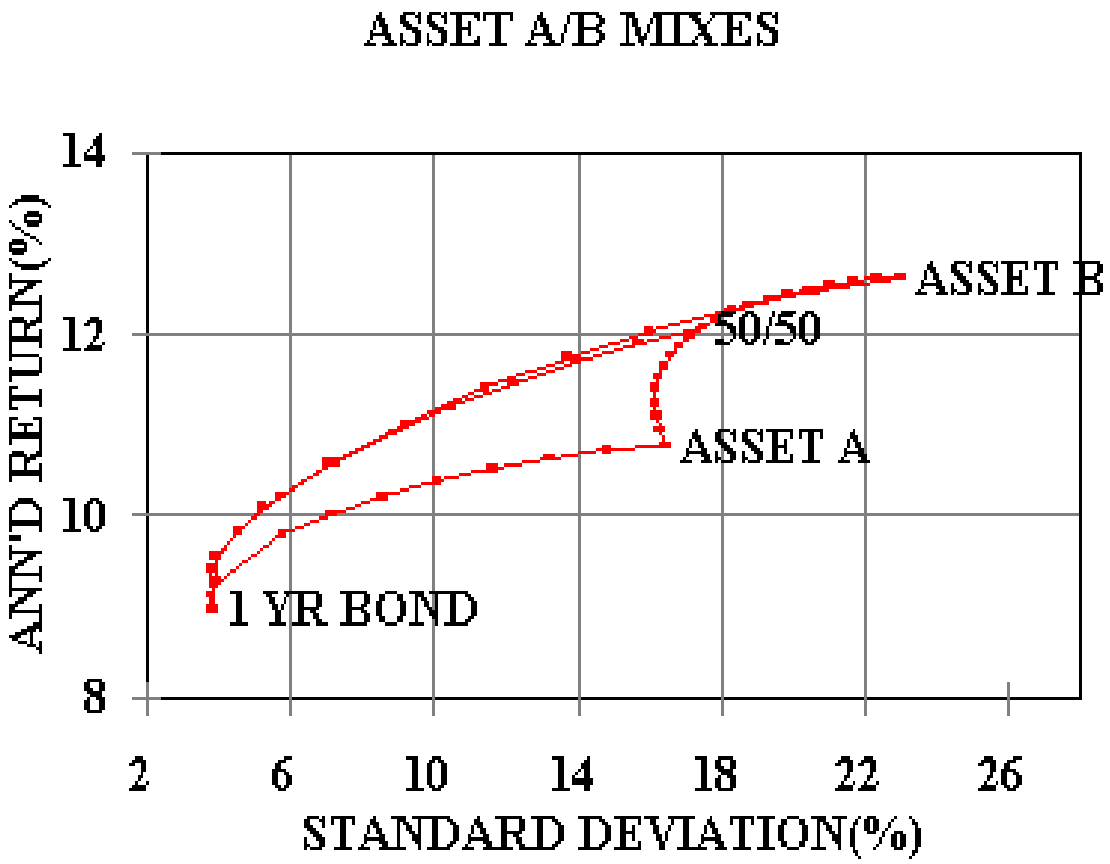
Consider this familiar Rorschach blot. It is a diagnostic tool most often used in the psychoanalysis of financial analysts:



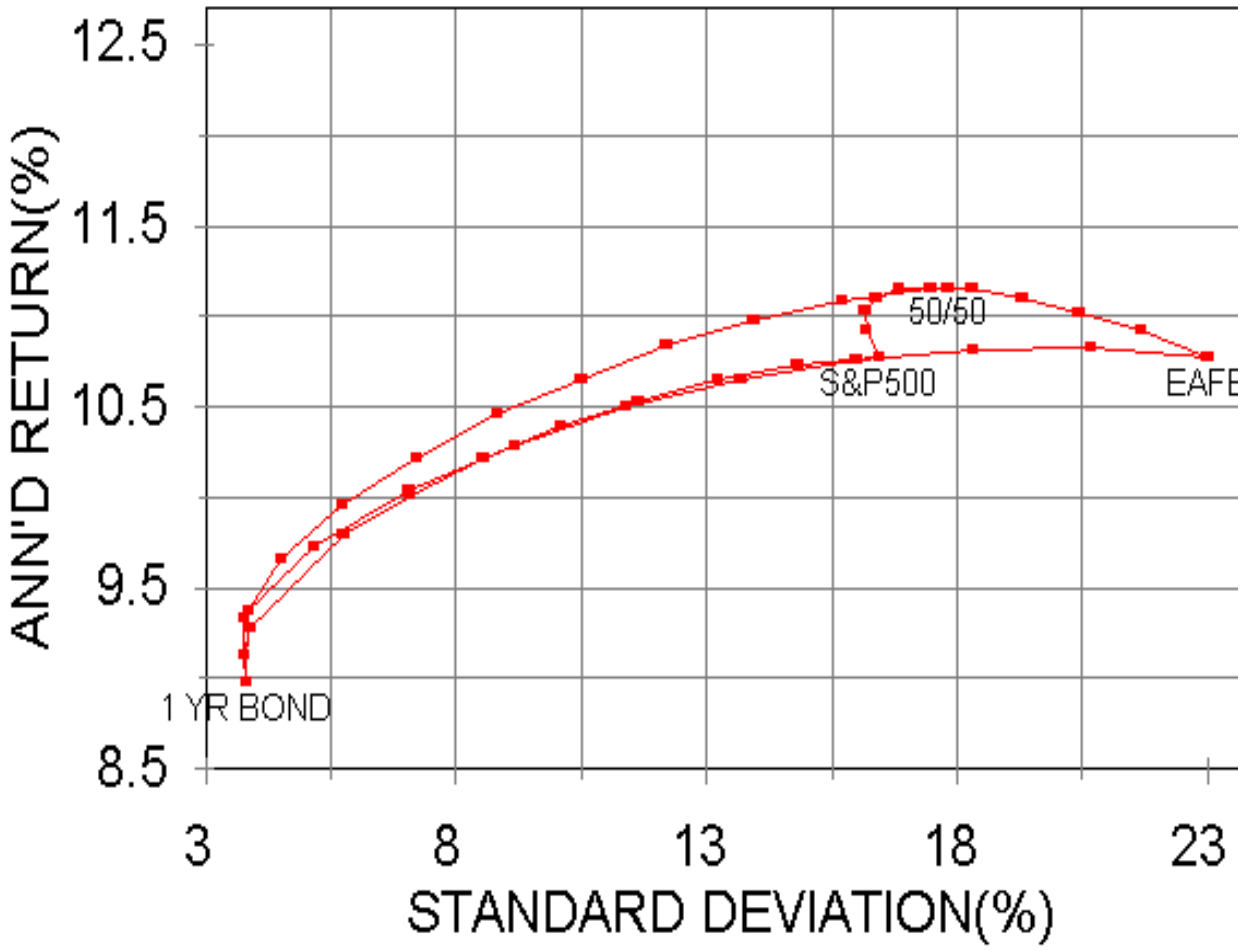
(The above graph covers the 1973-94 period, with annual portfolio rebalancing.) What does this hieroglyph tell us about the optimal disposition of one's equity assets between foreign and domestic securities? Not much, unfortunately. Portfolio backtesters and mean variance optimizers will still sneer at home market bias, while grayer heads will still worry about currency and sovereign risk. Purveyors of "value added" asset management will still continue to tout their unique ability to translate economic and political variables into a more profitable asset disposition. This famous risk/return plot is of only marginal use to those few who are 100% equity exposed -- most investors are considerably exposed to nonequity assets, most often bonds or cash equivalents. A more complete perspective is gained by considering the combination of various S&P/EAFE mixes with fixed income assets. In the below graph I have used DFA's 1 year fixed income corporate index to leaven the return/risk characteristics of various mixes of S&P/EAFE:



It is apparent from the above graph that during the 1973-94 period the addition of EAFE to the stock portion of one's mix produced salutary return/risk behavior up to a level of about 60/40 S&P/EAFE, above which no significant further benefit was obtained. However, extrapolating this data into one's current allocation requires extreme caution. For starters, the shape of the above plot is dominated by the higer return of the EAFE over the 1973-94 period. The 2% return advantage of the EAFE can be explained entirely by currency effects. It is doubtful that this currency advantage will persist into the indefinite future. Also, in 1973 Europe and Japan were still suffering from the lingering economic damage suffered in World War II, with living standards far below that of the US. By 1994 this differential no longer remained to be closed. Let's assume that it is January 1, 1973, and we are faced with the choice of allocating our resources between equities A and B, which have unknown outcomes, but will in retrospect turn out to be the same as the S&P and the EAFE. The cowardly investor might hedge his or her bets by allocating 50% to each:

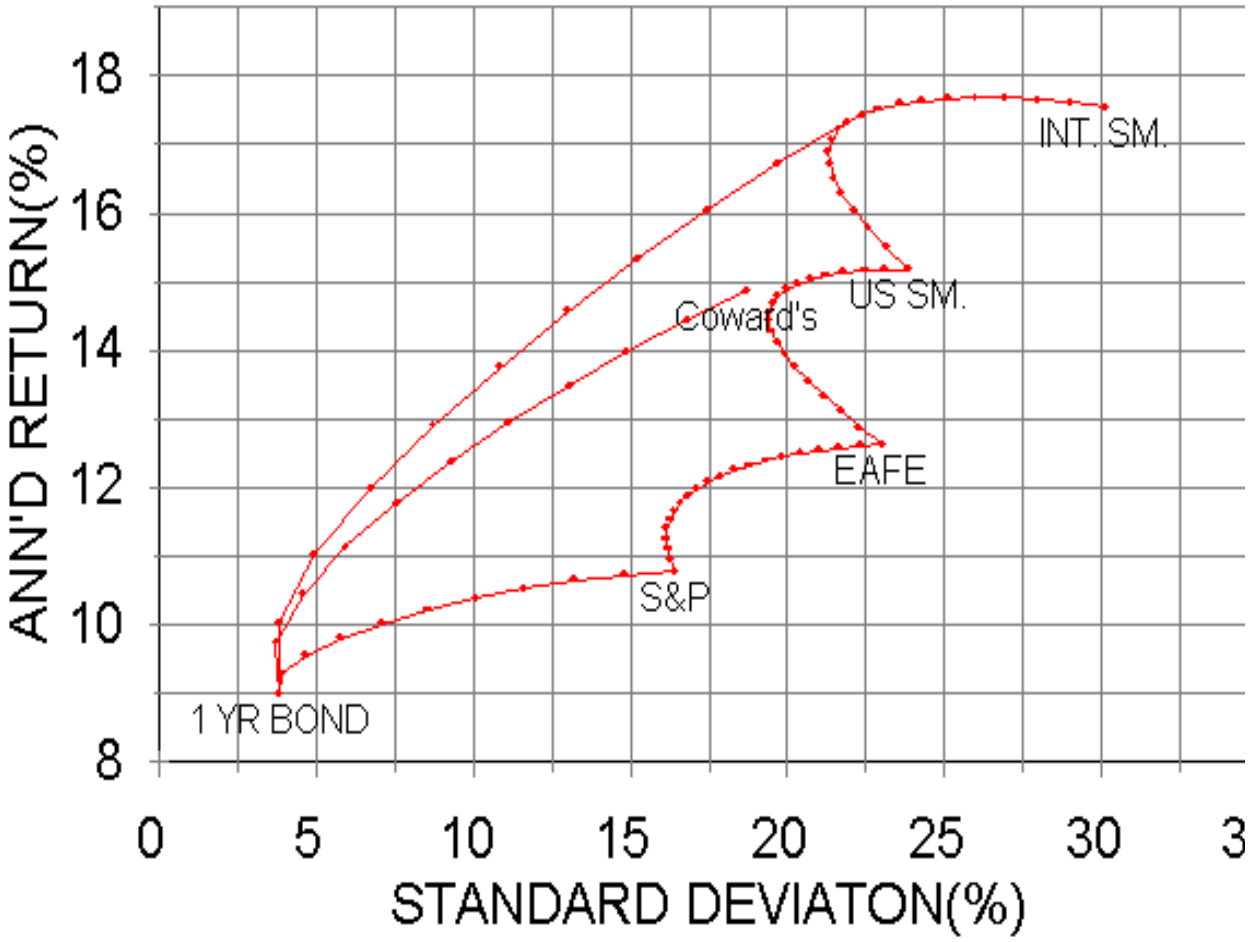


As you can see, this strategy worked quite well, with near optimal returns. Let's tweak the data further, and reduce the return of the EAFE to yield the same return as the S&P 500:



In this case, the "coward's portfolio" actually is significantly more efficient than either asset alone. The reason for this is simple: the annual returns on the EAFE and S&P 500 do not correlate perfectly, so addition of a small amount of EAFE results in both a decrease in risk as well as an increase in return (through rebalancing). It is important to understand that rebalancing is most effective when asset returns are similar, and that standard optimization techniques do not take rebalancing into account.

Over long time horizons small company stocks have provided higher returns than large company stocks. The below graph demonstrates that in fact for the 1973-94 period a 50/50 mix of US small stocks and international small stocks (one half each Japanese and UK) mixed with 1 year corporate bonds provided the most efficient portfolio mixes, yielding returns far higher than for S&P stocks alone, or S&P/EAFE mixes.There is no guarantee, of course, that small stocks will continue to outperform large stocks; most investors would be loathe to place all of their equity exposure in small stocks. A rational coward might split their equity exposure equally between S&P, EAFE, US small, and foreign small stocks. The below graph displays the plot for the 1973-94 period for the portfolio spectrum of this "coward's equity" mixed with 1 year corporates.



Once again, our coward did not do too badly -- risk adjusted returns are only slightly less than optimal, and much better than that obtained with the worst performing equity asset at each level of risk.

Optimizing a portfolio with historical returns, SDs, and correlations will never produce optimal future allocations. Similarly, the financial analytical equivalent of all of the king's horses and all the king's men will also fail to predict optimal future allocations, no matter how powerful the hardware, elegant the software, skilled the tailors, or impressive the verbal SAT scores. The best that can be hoped for is an allocation which will perform reasonably well under a wide variety of circumstances. The coward's approach seems to do this quite well.

The Coward's Equity Index (CEI)

When one can invest in foreign equities only as a single unit, a 50/50 foreign mix seems to work reasonably well. At present even very small US investors can invest in regional indexes and actively managed funds, so a higher portion of non US equity seems advisable. I thus propose the following "Coward's Equity Index:"

- 20% S&P 500
- 20% US small stocks (DFA US 9-10 Portfolio)
- 15% EAFE-Europe

- 5%EAFE Pac. Ex Japan
- 5% Japan Large (MSCI Japan)
- 10% Continental Small (DFA Cont. Sm. Co. Portfolio)
- 5%UK small (DFA UK Sm. Co. Portfolio
- 5% Japan Small (DFA Jap. Sm. Co. Portfolio)
- 5% Pac. EX Japan small (DFA Pac. Rim Sm. Co. Port., before 1/93 EAFE Pac. X J)
- 10% Latin American (MSCI Lat. Am.)

3 and 5 year returns and SDs, as well as return for the 7.5 year period from inception are available for both the index as well as the S&P500. (All returns are annualized, and the SD is calculated as twice the SD of quarterly returns.) As you can see, the Coward's Indexes have provided comparable returns and SDs to the S&P500, even though during this recent period the S&P considerably outperformed foreign stocks, particularly foreign small stocks.

Table 1 (Period Ending 6/96)

	3 Yr. Ret.	**3 Yr SD**	**5 Yr. Ret.**	**5 Yr. SD**	**8 Yr. Ret.**
100% CEI	16.54	6.74	16.07	6.85	14.18
100% S&P500	17.63	7.57	15.89	7.15	15.31
70%CEI 30%1 Yr. Corp.	13.00	4.83	12.86	4.99	12.04
70%S&P500 30%1 Yr. Corp.	13.74	5.67	12.73	5.31	12.77
40% CEI 60% 1 Yr. Corp.	9.50	2.98	9.67	3.00	9.79
40%S&P500 60%1 Yr. Corp.	9.91	3.77	9.60	3.50	10.18

The Small Investor's Coward's Equity Index (SICEI)

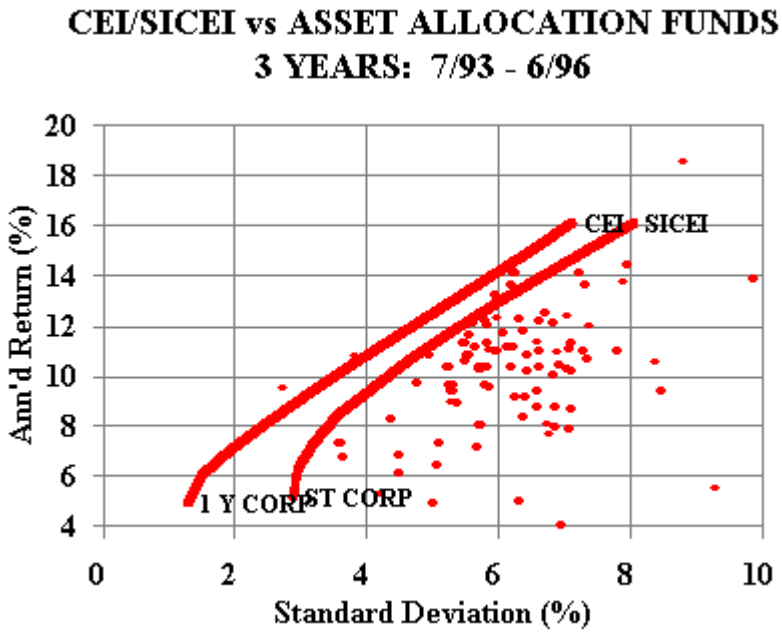
The Coward's Equity Index (CEI) can be nearly duplicated by using both of the leading index fund providers; the Vanguard and DFA organizatons. (Only the MSCI Latin American Index lacks an easily available corresponding index fund -- an activlely managed fund must be used for this region.) With a $3000 fund minimum, Vanguard is not a problem for most investors. DFA is another story -- their current investment minimum is $2,000,000, plus an approved financial advisor.

The CEI can be closely approximated by the small investor with the following fund allocation, which is known as the "Small Investor's Coward's Equity Index, " or "SICEI":

- 20% Vanguard Index Trust 500
- 20% Vanguard Small Cap Index Fund
- 15% Vanguard European Index Portfolio
- 7% Vanguard Pacific Index Portfolio
- 8% Vanguard Emerging Markets Index Portfolio
- 5% Scudder Latin America Fund
- 12.5% Tweedy Browne Global Value Fund
- 12.5% Acorn International Fund

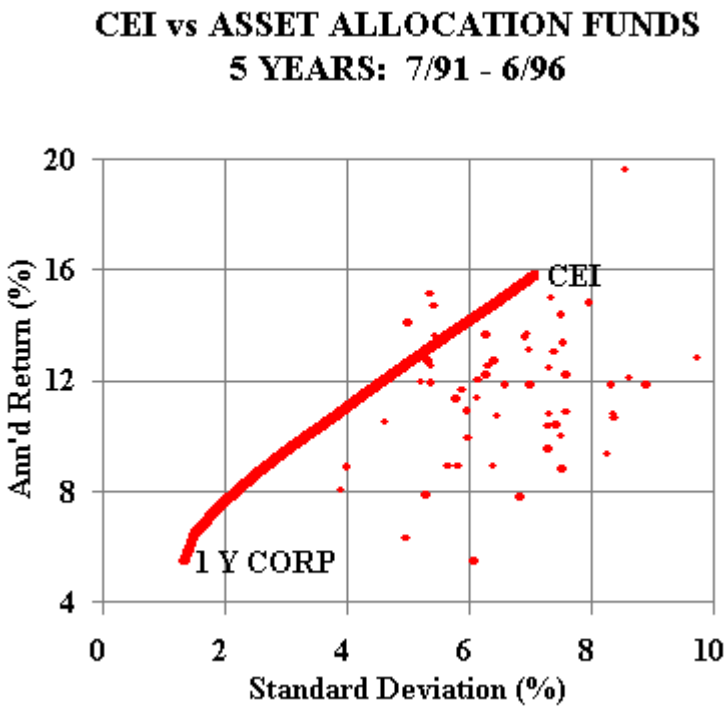
The equity portion of the Small Investor's CEI (SICEI) can be diluted with Vanguard's Short Term Corporate Bond Fund. The regional distribution of the SCEI is nearly identical to that of the CEI. The Acorn and Tweedy Browne funds were chosen to represent the entire foreign small cap component because of their differing characteristics. Acorn International has a high emerging market exposure, growth orientation, and is currency unhedged, while Tweedy Browne has a low emerging market exposure, value orientation, and is currency hedged.

I have plotted the 3 year returns and annualized SDs for the portfolio spectrum between 100% equity and 100% bond for both the CEI and the SICEI below:



The solid lines represent the behavior of the spectrum of mixes from 100% CEI and 100% SICEI to 100% bond. The other data points represent the 105 "asset allocation" and "global multiple asset" funds listed by Morningstar for the 3 year period.

The 5 year results are also plotted for the CEI (The SICEI does not yet have a 5 year track record.) and the 67 Morningstar "asset allocation" and "global multiple asset" funds listed for the period:



The Failure of Active Global Money Management

The 2 above graphs strikingly demonstrate that a naively constructed fixed asset allocation strategy will best the average institutional global asset allocator by about 3% per annum on a risk adjusted basis. There is nothing magic about the CEI and SICEI -- almost any reasonably balanced index based global asset allocation strategy will beat these bozos by a comparable margin over a long enough time period. Compare this with the 1% lag demonstrated by the average institutional domestic stock investor. One gets the impression that the more degrees of freedom given mutual fund managers, the worse they will perform. Allow them only to invest in domestic stocks, and their average results will be mediocre. Allow them to invest *anywhere in anything* and they will perform truly miserably. An apt analogy would be a class of first graders. Give them a task with simple and specific instructions, and it might get done. Tell them to do anything they like, and within a few minutes the classroom will look like a bomb hit it. I shall leave the explanation of this to others. I merely propose these portfolio spectra as benchmarks against which active global portfolio management can be bogied. (*Warning: The standard deviations for most equity assets for the past 3 and 5 year periods are considerably lower than for the previous decades. For planning purposes I suggest doubling the SDs displayed in the above 2 graphs.*)

I plan to make updates of the portfolio spectra for both the CEI and the SICEI a regular feature in ***Efficient Frontier***. Stay tuned.

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THE REBALANCING BONUS:

Theory and Practice

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ABSTRACT

The actual return of a rebalanced portfolio usually exceeds the expected return calculated from the weighted sum of the component expected returns. A formula for estimating this excess return is derived and tested. It is demonstrated that assets with high volatility and low correlation with the remainder of the portfolio provide considerable excess return, or "rebalancing bonus."

INTRODUCTION

Let's assume that you have decided that your desired portfolio composition consists of 50% S&P 500 stocks and 50% long term US treasury bonds. (If you've gotten this far in this website you probably already know that this is a fairly undesirable mix; it merely serves as a good example.) As these two assets frequently move in different directions, the portfolio will eventually drift from the desired 50/50 mix. If you desire to maintain the 50/50 composition, some of the better performing asset must be sold, and exchanged for a similar amount of the more poorly performing asset. This is known as *portfolio rebalancing*. An understanding of the mechanics of rebalancing is fundamental to sound portfolio management, and yet surprisingly little theoretical attention has been paid to this area. Three important questions concerning rebalancing arise:

1. Should rebalancing be done at fixed regular intervals ("periodically"), versus whenever the asset mix gets out of kilter by a certain amount ("threshold rebalancing")?

2. If done periodically, what is is the optimum interval (i.e., monthly versus quarterly versus annually)?

3. Is there a way to predict how much extra return is available from rebalancing, and is it possible to identify assets which provide superior rebalancing benefit?

For example, Markowitz[1] considers the return of a portfolio to be equal to the weighted sums of the individual component returns, but this formulation is valid only for nonrebalanced portfolios over single periods. It is surprising that Markowitz considered portfolio return to be

the weighted sum of the component returns, since he pioneered the idea that portfolio variance, or risk, is *not* the weighted sum of the individual component variances. In *Portfolio Selection* he presented Monte Carlo simulations in which the actual returns were significantly higher than the arithmetically derived expected returns, but did not comment on the discrepancy.

A simple, well known example will suffice: The return on common stock for the period 1926-94 was 10.19%, and for long term corporate bonds over the same period 5.51%.[5] The "Markowitz return" on an equal mixture of the two is the arithmetic mean, or 7.85%. Rebalancing this portfolio on an annual basis to maintain a 50/50 mixture yields a return of 8.34%; a "rebalancing bonus" of 0.49% is realized over the Markowitz return. In fact, however, if one had put equal amounts of money into stocks and bonds on January 1, 1926, and had not rebalanced or paid taxes, then the long term return would have been 9.17%. In this instance the nonrebalanced portfolio has a higher return than the rebalanced portfolio. This is because over the 69 year period studied the significantly higher stock return overwhelms the bond return; for the last 40 years of the period the unrebalanced portfolio consists of greater than 90% stock. Thus the higher return of the unrebalanced portfolio comes at the cost of a much higher risk than the rebalanced one. Perold and Sharpe[2] point out that rebalancing is a "concave" strategy. Portfolio insurance represents the opposite of rebalancing, and is referred to as a "convex" strategy. They suggest that convex portfolio insurance strategies as well as buy and hold ("flat") strategies produce superior returns in markets with a prolonged upward (or downward) bias, and concave rebalancing strategies produce superior returns in stagnant markets. If the only two assets considered are stocks and bonds, and if stock returns are always higher than bond returns over long time horizons, then obviously buy and hold, as well as portfolio insurance, will produce returns superior to rebalancing. As already pointed out, this will come at the cost of gradually increasing portfolio risk. However, things are very different when looking at global equity portfolios. Over very long time horizons there is usually relatively little difference in the returns in most national equity markets; under such circumstances rebalanced portfolios dominate. For example, when looking at the 1970-94 period, rebalancing various asset pairs almost always provides returns superior to nonrebalanced porfolios. Only when long term return differences among asssets exceed 5 percent do nonrebalanced portfolios provide superior returns, and then only at the cost of increased risk. (The exceptions which prove the rule are the very high returns of Japanese equity, and the very low returns of Australian and Italian equity.) Care should be taken to note that the superiority of rebalancing as a long term strategy pertains only to *national and regional* markets as a whole, and not among different industry groups. Over the course of decades entire industries often shrivel while others prosper mightily; rebalancing the pharmacutecal and petroleum segments of the S&P500 over the past three decades would have been a disastrous strategy. Although enitre national markets occasionally disappear throught war or nationalization, this occcurs much less frequently than the ongoing remolding of market segment capitalization characteristic of a market economy.

ESTIMATING THE REBALANCING BONUS

A small thought experiment is in order. Let us postulate a portfolio composed of equal parts of two assets, A and B. Further assume that each has a return of either +30% or -10%, with equal probability, and that the portfolio is rebalanced to 50/50 at the end of each year. The long term return of each asset is 8.1665% when annually compounded, and its standard deviation 20%. If

the annual returns are perfectly correlated (i.e., **r** =1) then the equal mixture of A and B will have the same risk and return as each individual asset, and no gain in risk or return is obtained by diversification. Now assume that there is a zero correlation between the returns of A and B. This can be symbolically represented by returns for four periods:

CASE 1

Asset	Year 1	Year 2	Year 3	Year 4
A	+30	+30	-10	-10
B	+30	-10	+30	-10
Equal Mix	+30	+10	+10	-10

This example yields an annualized return of 9.0794%, and a standard deviation of 14.142%. The standard deviation of the portfolio has been reduced by a factor of the square root of 2 (and the variance halved) as predicted by Markowitz. However, the return is 0.9129% higher than the Markowitz return. Further, this excess return would not have been realized without rebalancing. This portfolio thus has a "rebalancing bonus" (hereafter known as the "RB") of 0.9129%. If instead the returns on A and B are perfectly inversely correlated (**r** = -1) it can be seen that the return of the portfolio each year will be 10%, as will be the annualized return, and the standard deviation and variance zero. The RB will thus be 1.8355% in this highly theoretical instance, which is slightly more than twice the bonus in the instance where **r** =0. Simple inspection suggests that the RB is proportional to:

(1 - correlation coefficient)

Assume that the returns on asset A are still +30 and -10, but that now the return on B can be either +15 or +5. Asset B then has an expected return of 9.8863%(and a standard deviation of 10%; one half of the SD of asset A). The Markowitz return for a 50/50 mixture of A and B is 9.0264% (the average of 8.1665% and 9.8863%).

For the **r** =0 uncorrelated example:

Case 2

Asset	Year 1	Year 2	Year 3	Year 4
A	+30	+30	-10	-10
B	+15	+5	+15	+5
Equal Mix	+22.5	+17.5	+2.5	-2.5

Here, the annualized return is 9.5155%, yielding an RB of 0.4891%, or about one half of the uncorrelated example in case 1. Further experimentation demonstrates that the rebalancing bonus of any combination of two assets is roughly approximated by the expression:

$$X_1X_2SD_1SD_2(1 - \text{correlation coefficient})$$

where SD_1 is the standard deviation of asset 1, X_1 is the amount of asset 1.

Taking this process one step further, consider the case where an asset with returns of either +30 or -10 is mixed with a riskless asset whose return is always zero. The return on a 50/50 mix of these two assets will be either +15 or -5. The Markowitz return of this mix is 4.0833% (the mean of 8.1665 and zero), but it's actual return is 4.5227%, yielding an RB of 0.4394%. Since the standard deviation of the riskless asset is zero, the expression derived above will

equal zero also, so an additional term is needed. A bit more experimentation shows that this term can be closely approximated by:

$$X_1X_2(SD_1\text{-}SD_2)^2/2$$

Thus, it is postulated that the "theoretical RB," $RB_{1,2}$, can be expressed as:

$$RB_{1,2} = X_1X_2\{SD_1SD_2(1\text{- C.C.}) + (SD_1 - SD_2)^2/2\}$$

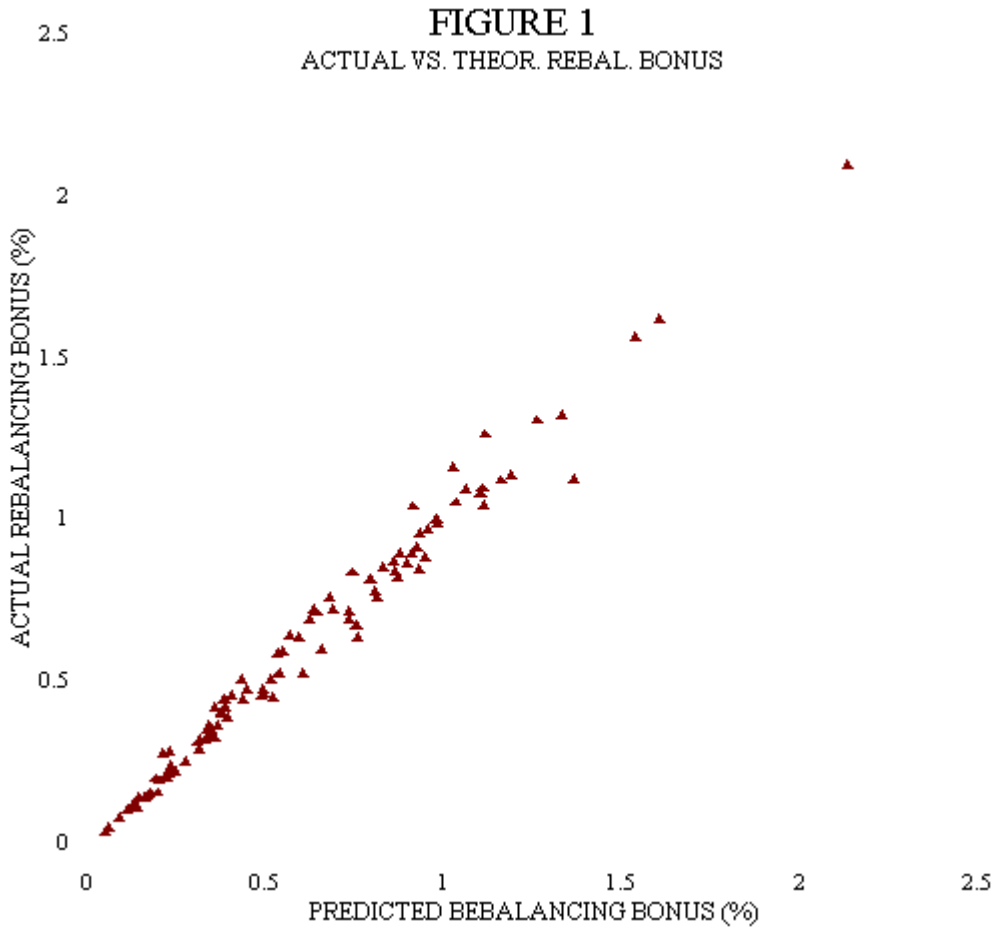
or more simply,

$$X_1X_2(Var_1/2 + Var_2/2 - Covar_{1,2})$$

Stated succinctly, *the intrinsic rebalancing potential of any asset pair is the difference between its mean variance and covariance.* The above formulation was tested for 50/50 mixtures of asset pairs of the below assets:[5]

S&P 500 index
MCSI-EAFE
US 9-10 portfolio (DFA US small co. portfolio)
Equity REIT(NAREIT index)
MSCI-EAFE Europe
MSCI-EAFE Pacific
MSCI-EAFE Emerging Markets
UK small stocks(DFA UK small co. portfolio)
Japanese small stocks(DFA Japanese small co. portfolio)
Continental European small stocks(DFA Continental small co. portfolio)
Precious Metals(Morningstar fund index)
Natural Resources(Morningstar fund index)
Lehman Long Term Government Index
International Bond(Morningstar fund index)
One year corporate notes(DFA one year fixed fund)

The theoretical RB was calculated from SDs and correlation coefficients derived from quarterly data for the period July 1988 to December 1994. Similarly, the "actual" portfolios were rebalanced quarterly. A linear correction was applied to each series of quarterly returns to yield a zero annualized return for each individual asset over the whole study period; thus any return obtained from the asset pair must be due to rebalancing.



The 105 resultant theoretical/actual data points for each asset pair were plotted in Figure 1. As is seen, the theoretical data is highly predictive of the actual data, with an R^2 of 0.983 and a standard error of 0.052% of yield.

To summarize thus far: The beneficial effect of poorly correlated assets on portfolio risk, undoubtedly appreciated since antiquity, was mathematically formalized only forty years ago by Markowitz. The above theoretical model and backtested portfolios suggest that significant excess return is available from combinations of asset pairs which have both low correlation and high risk.

The capital asset pricing model asserts that the "risky" portion of one's portfolio should be a capitalization weighted "market portfolio" of all the world's investible risky assets.[3] The above considerations provide an argument against portfolio capitalization weighting, since by definition capitalization weighting denies the need for rebalancing. Assets and sectors which have been battered or inflated will find their capitalization weighting commensurately reduced or increased. Consider the MSCI-EAFE index. During the study period July 1988 to December 1994 the US investor passively employing this index saw their percentage of Japanese holdings increase and decrease precipitously. On the other hand, the investor who kept his or her foreign assets equally divided between the European and Pacific sides of this index reaped an excess return of 0.588% for the period studied, with a modest concomitant reduction in risk. The emerging markets provide an even more vivid example of this phenomenon. The Latin American and Far Eastern components of the MSCI-EAFE EM index have very high standard deviations and are imperfectly correlated. The above model suggests that the investor

who actively rebalances these two sectors will reap an advantage of at least two or three percent over their passively invested colleague. It seems highly likely that adherence to fixed percentage allocations among finely divided market sectors offers significant advantages over a passive market portfolio/market capitalization approach.

FREQUENCY OF REBALANCING

Although the need for portfolio rebalancing to policy composition is generally acknowledged, scant attention has been paid to it's frequency. In a rare paper addressing this issue, Arnott and Lovell [4] found that for the years 1968-91 a 50/50 stock/bond mix produced a return of 9.02% for annual rebalancing, 9.12% for quarterly rebalancing, and 9.16% for monthly rebalancing. Trading costs reduced the rebalancing benefit slightly, but even monthly rebalancing was still of benefit after transaction costs were considered. They also found that nonperiodic "threshold rebalancing" when portfolio composition surpassed 1%, 2%, and 5% excess/deficiency produced somewhat lesser gains. They surmised "Over this period, regular monthly rebalancing returns dominated less active approaches. Should one infer that daily rebalancing is better still? One cannot say, but it seems plausible."

The above formulation provides a framework for further investigation of the rebalancing frequency problem. One can calculate a "theoretical rebalancing bonus" for each of the different periods. It is well known that asset return correlations vary somewhat over time. It is less well appreciated that the individual return standard deviations and paired correlation coefficients are not the same for differing sampling intervals within the same epoch. For a given historical epoch the standard deviation of returns of a single asset may be different for shorter or longer measuring intervals. Even larger differences may be present for correlation coefficients of asset pairs. To illustrate this point, and to provide a basis for further discussion, I have provided standard deviation and correlation data for five different assets for monthly, quarterly, and annual returns in Table 1: UK, Japanese, Continental, and US 9-10 decile small stocks, and the MSCI-Emerging Markets Index, again for the July 1988-December 1994 period. These particular assets were chosen because of their low mutual correlations and high return standard deviations.

Next, "actual" or "observed" RBs for the resultant 10 asset pairs were calculated for monthly, quarterly, and annual rebalancing, and compared to the "theoretical," or "predicted" value. The data are presented in Table 2.

Table 1

	Standard Deviation(%)	**CONT SM**	**JAP SM**	**MSCI-EM**	**US SM**	**UK SM**
CONT SM	m: 14.54 q: 15.87 a: 19.86	m: 1.000 q: 1.000 a: 1.000				
JAP SM	m: 32.29 q: 28.57 a: 24.63	m: .539 q: .483 a: .798	m: 1.000 q: 1.000 a: 1.000			
MSCI-EM	m: 22.57 q: 27.06 a: 30.96	m: .154 q: .123 a: .634	m: .244 q: -.098 a: .653	m: 1.000 q: 1.000 a: 1.000		
	m: 14.71	m: .099	m: .156	m: .379	m: 1.000	

US SM	q: 18.25 a: 19.50	q: .007 a: -.154	q: .217 a: .242	q: .552 a: .590	q: 1.000 a: 1.000	
UK SM	m: 19.91 q: 17.79 a: 13.82	m: .620 q: .478 a: .251	m: .541 q: .478 a: .378	m: .305 q: .158 a: .614	m: .302 q: .309 a: .424	m: 1.000 q: 1.000 a: 1.000

(m=monthly, q=quarterly, a=annual All SDs are annualized, calculated as twice the quaterly SD, and 3.4641 times the monthly SD)

Table 2

Asset Pair	Monthly RB actual/predicted	Quarterly RB actual/predicted	Annual RB actual/predicted
CONT SM/JAP SM	0.911/0.871	0.818/0.787	0.345/0.275
CONT SM/MSCI-EM	0.788/0.778	1.099/1.102	0.941/0.717
CONT SM/US SM	0.484/0.482	0.694/0.762	1.171/1.118
CONT SM/UK SM	0.311/0.311	0.412/0.369	0.529/0.551
JAP SM/MSCI-EM	1.455/1.469	2.099/2.125	0.681/0.712
JAP SM/US SM	1.347/1.388	1.124/1.153	0.929/0.943
JAP SM/UK SM	0.917/0.930	0.885/0.940	0.710/0.678
MSCI-EM/US SM	0.591/0.592	0.600/0.650	0.705/0.783
MSCI-EM/UK SM	0.787/0.790	1.267/1.109	0.987/0.779
UK SM/US SM	0.541/0.545	0.644/0.651	0.477/0.428

These data make three main points:

1) No one rebalancing period dominates. Monthly rebalancing was best in three cases, quarterly in four, and annual in three.

2) The theoretical RB was highly predictive in all cases where significantly different rebalancing bonuses were obtained with different intervals. Only in three instances were minor (3) Those who rely on optimization procedures must be cognizant of the sampling interval used to calculate the standard deviation and correlation input data. Optimization outputs are liable to be very different for data obtained from monthly, quaterly, and annual sampling from within the same historical period.

Obviously, real portfolios are far more complex than asset pairs. It seems reasonable to assume that, analogous to the Markowitz formula for portfolio variance, the formula for a portfolio RB can be represented by:

$$\text{½}(\text{SUM}_{i=1..n})(\text{SUM}_{j=1..n})\text{RB}_{ij}$$

(Each term for which i=j will be zero.) Applying this formula to an equal mix (20% each) of the five assets used above yields an observed/predicted RB of 1.205/1.118% for annual, 1.542/1.556% for quarterly, and 1.304/1.301% for monthly rebalancing. The annualized portfolio SDs are 16.80% for annual, 13.65% for quarterly, and 14.55% for monthly rebalancing--the same preference ranking as for the RB. This should not be surprising as asset pair covariance is the dominant component for both portfolio RB and portfolio variance for

porfolios where n>2. Further work is necessary to evaluate the value of this formula for the estimation of rebalancing benefit for more complex portfolios. It may transpire that the complex relationship between pair correlations and single asset standard deviations measured over shorter or longer time period lengths will not be stable enough to render ex post analysis of much value.

PRECIOUS METALS STOCKS -- A SPECIAL CASE

The portfolio characteristics of precious metals equity are unique; very low long term return, very high return variance, and near zero correlation with most other asset classes. One of the primary rationales for this behavior is that most of the risk of this asset class is nonsystematic because of its low correlation with other assets -- in other words, it can be diversified away. The above discussion provides another perspective on this paradox. Examination of the theoretical rebalancing formula shows that the addition of a small amount of a high variance zero correlating asset to a portfolio with a much lower variance increases its apparent return by approximately one half of its variance. In other words, since the variance of a typical portfolio of precious metals stocks is about 0.1, its apparent return (IRR) in a rebalanced portfolio will be about 5% higher than its long term stand alone return. This is precisely what is observed by the investor who periodically rebalances the precious metals component of their portfolio as a fixed proportion; a large fraction of the IRR of this component comes from rebalancing per se. Thus, not only is the systematic risk of precioius metals stocks much lower than its stand alone risk, but its rebalanced portfolio return is much higher than its observed stand alone long term return.

CONCLUSIONS:

1. The expected return of a rebalanced portfolio is not accurately represented by simple arithmetic weighting of individual asset returns. This is particularly true of assets which have high standard deviations and are poorly correlated. This may be of some importance to those utilizing optimization technology which depends on linear expected return determinations. Standard MVO techniques underestimate the benefit of high variance/low correlation assets.

2. The capitalization weighting suggested by the CAPM does not appear to be maximally efficient.

3. A formula is presented which accurately predicts rebalancing benefit. The inputs to this equation are portfolio composition, individual asset standard deviation/variance, and asset pair correlation/covariance. Since these parameters vary with sampling interval, it may be possible to use this formula to predict the optimal rebalancing frequency. Further, the results of the analyses presented suggest that shorter rebalancing intervals may not always be optimal.

4. Optimizers should understand that sampling interval will significantly affect both input and output data, and that he or she is optimizing only for standard deviatons (or other risk measures, such as semivariance) over a given sampling interval.

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3. Roger G. Ibbotson and Gary P. Brinson, *Global Investing: the Professional's Guide to the World Capital Markets* (New York: McGraw Hill Inc., 1992), 43-44.

4. Robert D. Arnott and Robert M. Lovell, "Rebalancing: Why? When? How Often?" *The Journal of Investing* 2, no. 1 (Spring 1993): 5.

5. US stock and bond returns from *Stocks, Bonds, Bills and Inflation 1995 Yearbook*, Ibbotson Associates, Inc., Chicago. The 1988-94 quarterly returns data were obtained from *Principia* for closed and open end funds from Morningstar Associates, Chicago.

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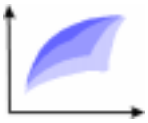

Roll Your Own

Become Your Own Portfolio Analyst

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This piece is intended for those who desire to perform their own portfolio analysis. If you are even marginally computer literate, this is not as difficult as you might think. If you are familiar with spreadsheets, it's downright simple.

MEAN VARIANCE OPTIMIZATION

The classic method of portfolio analysis involves "mean variance optimization" ("MVO"), a complex mathematical technique which calculates optimal portfolio compositions. The input data are the returns and standard deviations for each asset, as well as the correlations among each possible asset pair. For example, a portfolio with 4 possible assets requires 14 bits of data -- 4 returns, 4 SDs, and the 6 unique correlations possible among the 4 assets. A portfolio of 10 possible assets requires 65 individual bits of data -- 10 returns, 10 SDs, and the 45 unique correlations between the 10 assets. Most commercially available optimizers are ridiculously expensive -- at least $500 for the most basic models, not including input data. The one exception is the optimizer available from Portfolio Software. Their DOS based programs are quite clunky, but they do the job for $150 (Actually, a year ago they could be had for $35 -- and we all thought inflation was dead).

One major problem with MVOs is that their return/SD/correlation inputs are not easily come by -- you must be reasonably familiar with this sort of data. A good source of basic return/SD/correlation data can be found in *Global Investing* by Gary Brinson and Roger Ibbotson. Another major problem with MVO is that it does not take the benefit of portfolio rebalancing into effect, which is a significant shortcoming when dealing with high risk assets such as international small cap, emerging markets, and precious metals stocks.

SPREADSHEET BASED PORTFOLIO BACKTESTING

A simpler and potentially more powerful technique for portfolio analysis involves the construction of spreadsheets which calculate the returns and SDs of different asset mixes. All that is required is a conventional spreadsheet program such as Lotus 123, Quattro Pro, or Microsoft Excel, and historical return data. This method has 3 major advantages over MVO:

1. The input data is simpler and more readily available.
2. The technique automatically includes rebalancing.
3. With some experience you can employ your own custom measures of risk.

The only real difficulties with spreadsheet analysis are that the optimizers resident in the spreadsheets can be a bit balky, and that it is difficult (but not impossible) to vary asset returns.

I've made a sample template for **Excel** for the 1926 Ibbotson data. (Note: This file is not really a .zip file. Do not bother to try to unzip it. It will run as is in your Excel. You might want to rename or save it as an .xls file.)

Because of fair use restrictions I'm not willing to include the required annual return input data, but this is easily available from *Stocks, Bonds, Bills, and Inflation* from Ibbotson Associates. You can buy their yearbook for $95, or you can peruse your local library's *SBBI* annual return page. The annual returns data is entered in blocks C2..BT6. This will be a bit tedious, but should take no more than 30 minutes of your time. Since annual data is used, annual rebalancing is assumed. You can devise a monthly returns spreadsheet if you wish, but this will require entering over 4000 bits of data! To get up and running,

1. Download the apropriate file.

2. Obtain and enter the returns data in blocks C2..BT6.

3. Enter the desired portfolio compositions in blocks B8..B12 as decimals (e.g., 55% = 0.55). Make sure that all of your compositions add up to 1.0 in block B15. The number of study years should be entered in block B28 (70 for the period 1926-95).

4. Read the annual returns in blocks B13..BT13, the wealth index (growth of $1 invested in 1926) in block B16, the annualized total return in block B17, and the SD of the portfolio in block B18.

5. Trailing 3, 5, 10, 20, and 30 year returns may be read in rows 23, 24,25,26, and 27, respectively.

Good luck. Future issues of ***Efficient Frontier*** will describe more sophisticated models.

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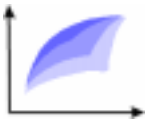

DATAQUEST

The Search for Usable Time Series

Probably the most difficult task facing the amateur portfolio theorist is the garnering of reliable asset returns data appropriate for use in mean variance oprimizers and backtesters. There are numerous sources of time series data available on the web (see in particular the Federal Reserve's FRED, J. P. Morgan's RiskMetrics, and FINWEB's Financial Data Finder.) Unfortunately, almost none of these are in the form of usable monthly, quarterly, or annual *total return* data for broad asset classes. Webwise, the only really usable publically available time series I'm aware of are the Vanguard Index Funds and the Barra series, which are fairly limited in both time and scope. Both of these are most easily accessed through Bill Sharpe's website.

The most useful publically available total return time series are available from Morningstar and Ibbotson. Unfortunately, ***Efficient Frontier*** is unable to reproduce these data without violating copyright law.

Efficient Frontier would greatly appreciate contact from anybody who is aware of the existence of other usable public domain asset class total return time series data, or from any institutional players who are willing to make their data public. You can reach ***Efficient Frontier*** at wbern@mail.coos.or.us.

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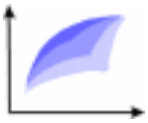

Dataquest of the Month

Does anybody out there have a reliable precious metals equity time series?

The price of gold, silver, and other precious metals going back decades is relatively easily obtained from a number of sources, such as the Federal Reserve Bank (FRED). More fragmentary longer term data (going back centuries) is also available (see, for example *Global Investing* by Gary Brinson and Roger Ibbotson).

Most investors, large and small, make only limited use of metal coins and bars in their portfolios -- the most favored precious metals investment vehicles are mining company stocks and the mutual funds which invest in them. Aside from Morningstar's mutual fund database, which constitutes only a limited number of funds going back as far as 1976, I'm not aware of a time series for a reliable index of precious metals equities. Is anyone aware of reliable *total return* time series data for precious metals equity, preferably derived from COMPUSTAT or CRSP data?

If you are, please contact ***Efficient Frontier*** at wbern@mail.coos.or.us.

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Bequeathing Your Assets to Your Broker

(Thank You, Paine Weber)

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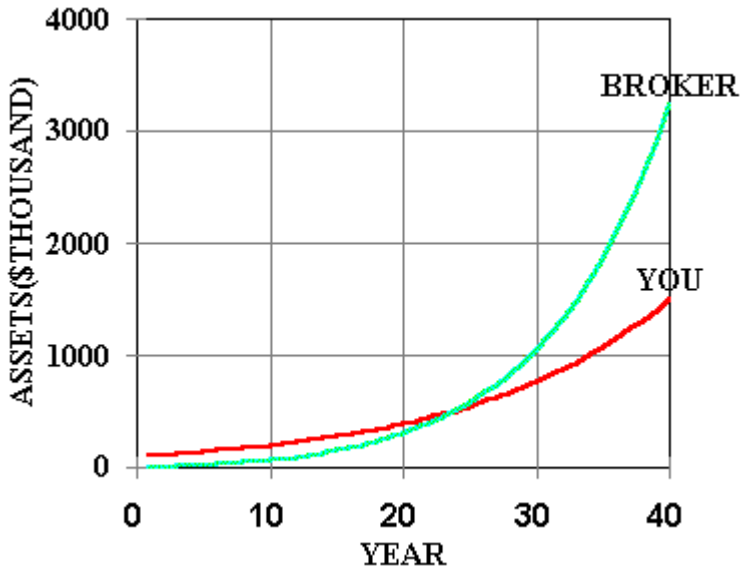
A hoary peice of stockbroker lore has a young broker asking a senior partner at the firm about his proudest accomplishment. The reply:

> "Over the years I've gradually transferred the assets of my clients to my own name."

Awaredness of investing expense has heightened in recent years, and it is instructive to examine the truth of this old joke. Imagine for a moment that you inherited $100,000 at age 25. Not long after an enterprising broker at a "full service" firm calls and offers you his services. He is quite personable and always seems to have a snappy explanation for the behavior of the market; you engage his services. Considering your conservatism, he invests your portfolio in a 50/50 mix of common stocks and bonds. What can you expect in the way of long term return? What will become of his commision stream? The first question is easy to answer. Assuming that he is of average ability, this mix will produce a long term return of about 8% . (The long term return of common stocks is about 10%-11%, of long term corporates about 5%-6%.).

Unfortunately, *your* return will not be that high. Whether you use a "wrap" account or employ a straight comission basis, fees will reduce your returns by approximately 3% per year -- to about 5%. This is only 2% greater than the long term inflation rate of 3%.

This is discouraging enough. Now consider what the broker accomplishes with your comissions. He now has a steady income stream, consisting of 3% of your assets each and every year, to invest. He should also earn the same 8% return, but his investment return will not be substantially reduced by comissions and fees.



The above graph shows that in 24 years the brokerage will have parlayed your comissions into a sum equal to your own. When you retire at 65 you will have amassed $704,000. Your broker has done far better; he has produced $1,542,000 with the comissions from your account.

The above calculation is somewhat artificial. The grim reality is actually much worse. Since your broker will be turning over the assets in your account with some regularity to earn comissions and/or justify their fees, this will generate significant capital gains. Taxes on these gains, as well as your bond coupons, will reduce your investment yield about 3%, just keeping up with inflation. In contrast the broker will most likely manage his assets with little or no turnover, further widening his asset advantage.

For the full service brokerage customer, then, the old broker's remark is no joke -- eventually, your broker will wind up with more of your own assets than you do. For the no load fund investor, things are a little brighter, but still fairly grim. Assume Fidelity charges you 1% in expenses to manage your portfolio. Under the above 40 year scenario, this leaves you with $1,479,000, but Uncle Ned still earns $722,000 for himself. Vanguard should be able to invest your assets in their index funds for about 0.25% -- the numbers here are $1,980,000 for you, $169,000 for them.

Obviously, Paine Weber, Fidelity, and Vanguard do not get to keep *all* of their expenses. Fidelity will use a large part of their management fees for advertising, enabling them to manage an even greater portion of the wealth of Western Civilization. Vanguard's expenses are so low that it is doubtful that there is much profit margin. In any case, Vanguard is actually owned by its funds' shareholders; John Bogle is not getting terribly rich at your expense.

So, keep an eye on those expenses. Thank you, Paine Weber (Merrill Lynch, Smith Barney, etc., etc.).

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